

On Resonant Leptogenesis

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Abstract. It has been recently shown that the quantum Boltzmann equations may be relevant for the leptogenesis scenario. In particular, they lead to a time-dependent CP asymmetry which depends upon the previous dynamics of the system. This memory effect in the CP asymmetry is particularly important in resonant leptogenesis where the asymmetry is generated by the decays of nearly mass-degenerate right-handed neutrinos. We study the impact of the nontrivial time evolution of the CP asymmetry in resonant leptogenesis, both in the one-flavour case and with flavour effects included. We show that significant qualitative and quantitative differences arise with respect to the case in which the time dependence of the CP asymmetry is neglected.

1. Introduction

Thermal leptogenesis [1, 2, 3] is a simple mechanism to explain the observed baryon number asymmetry (per entropy density) of the Universe $Y_B = (0.87 \pm 0.02) \times 10^{-10}$ [4]. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B + L)$ -violating sphaleron interactions [5, 6] which exist in the Standard Model (SM). A simple model in which this mechanism can be implemented consists of the SM plus three right-handed (RH) Majorana neutrinos. In thermal leptogenesis [2] the heavy RH neutrinos are produced by thermal scatterings after inflation and subsequently decay out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov's constraints [7, 6]. RH neutrinos are also a key ingredient in the formulation of the well-known “see-saw” (type I) mechanism [8], which explains why neutrinos are massive and mix among each other and why they turn out to be much lighter than the other known fermions of the SM. For such a reason thermal leptogenesis has been the subject of intense research activity in the last few years with the main goal of providing a quantitative relation between the light neutrino properties and the final baryon asymmetry.

Thermal leptogenesis is based on the assumption that RH neutrinos are efficiently generated by thermal scatterings during the reheating stage after inflation. In the scenario in which the RH neutrinos are hierarchical in mass, successful leptogenesis requires that the RH neutrinos are heavier than about 10^9 GeV [9]. Hence, the required reheating temperature cannot be much lower [2, 10, 11, 12]. In supersymmetric scenarios this may be in conflict with the upper bound on the reheating temperature necessary to avoid the overproduction of gravitinos during reheating [13]. Indeed, being only gravitationally coupled to the SM particles, gravitinos may decay very late jeopardising the successful predictions of Big Bang nucleosynthesis.

In the resonant leptogenesis scenario [14, 15] this tension may be avoided. If the RH neutrinos are nearly degenerate in mass, self-energy contributions to the CP asymmetries may be resonantly enhanced, thus making thermal leptogenesis viable at temperatures as low as the TeV. Resonant leptogenesis seems to us a natural possibility. In the absence of simple predictive models one expects that left-handed and RH neutrinos show similar levels of degeneracy. Indeed, within the see-saw mechanism, quasi-degenerate light neutrinos are more naturally explained by quasi-degenerate RH neutrinos, rather than by an interplay between Yukawa couplings and the masses of the RH neutrinos. The quasi-degeneracy in the RH sector can be easily explained by symmetry arguments, *e.g.*

a slightly broken $SO(3)$ symmetry.

Resonant leptogenesis is the subject of the present paper. More specifically, we will investigate the resonant leptogenesis scenario in view of the recent results achieved by studying the dynamics of thermal leptogenesis by means of quantum Boltzmann equations [16] (for an earlier study, see Ref. [17]).

Let us pause here for a moment and summarize why quantum Boltzmann equations are relevant in resonant leptogenesis [†]. The generation of the baryon asymmetry occurs when RH neutrinos are out-of-equilibrium. Therefore, their abundance and the one for the lepton asymmetry are determined by Boltzmann equations. In the classical Boltzmann equation approach, every scattering in the plasma is independent from the previous one and the particle abundances at a given time do not depend upon the previous dynamical history of the system. Quantum Boltzmann equations are obtained starting from the non-equilibrium quantum field theory based on the Closed Time-Path (CTP) formulation. It is a powerful Green's function formulation for describing non-equilibrium phenomena in field theory. It allows to obtain a self-consistent set of quantum Boltzmann equations for the quantum averages of operators, *e.g.* the lepton asymmetry operator, evaluated in the in-state without specifying the out-state. The quantum Boltzmann equations have an obvious interpretation in terms of gain and loss processes. What is unusual, however, is the presence of the integral over time in the scattering terms where theta functions ensure that the dynamics is causal. The quantum Boltzmann equations are therefore manifestly non-Markovian. Only the assumption that the relaxation timescale of the particle asymmetry is much longer than the timescale of the non-local kernels leads to a Markovian description. A further approximation, *i.e.* taking the upper limit of the time integral to infinity, leads to the familiar classical Boltzmann equation. The physical interpretation of the integral over the past history of the system is straightforward: it leads to the typical “memory” effects which are observed in quantum transport theory [18, 19]. The thermalization rate obtained from the quantum transport theory may be substantially longer than the one obtained from the classical kinetic theory.

Furthermore, and more importantly, the CP asymmetry turns out to be a function of time, even after taking the Markovian limit. Its value at a given instant depends upon the previous history of the system. If the time variation of the CP asymmetry is shorter than the relaxation time of the particles abundances, the solutions to the quantum and

[†] For more technical details the reader is referred to Ref. [16].

the classical Boltzmann equations are expected to differ only by terms of the order of the ratio of the timescale of the CP asymmetry to the relaxation timescale of the distribution. In thermal leptogenesis with hierarchical RH neutrinos this is typically the case. However, in the resonant leptogenesis scenario, at least two RH neutrinos N_1 and N_2 are almost degenerate in mass and the CP asymmetries from the decays of the RH neutrinos are resonantly enhanced if the mass difference $\Delta M = (M_2 - M_1)$ is of the order of the decay rates. The typical timescale to build up coherently the CP asymmetry is of the order of $1/\Delta M$, which can be larger than the timescale for the change of the abundance of the RH neutrinos. This tells us that in the case of resonant leptogenesis significant differences are expected between the classical and the quantum approach.

The rest of the paper is organized as follows. In Section 2 we study the impact of the time-dependent CP asymmetry on the final lepton asymmetry, in the one-flavour approximation. The numerical results are supported by analytical estimates, for both the regimes of strong and weak wash-out. The weak wash-out case is analyzed in greater detail since this is the regime where the time dependence of the CP asymmetry becomes more relevant. The generalization to more than one flavour is given in Section 3, where we discuss the possible wash-out regimes. Again, the analytical formulae confirm the numerical solutions. Finally, Section 4 contains our conclusions.

2. Resonant leptogenesis revisited: the one-flavour case

Our starting point is the SM plus three RH neutrinos N_α ($\alpha = 1, 2, 3$), with Majorana masses M_α . The interactions among RH neutrinos, Higgs doublets H , lepton doublets ℓ_i and singlets e_i ($i = e, \mu, \tau$) are described by the Lagrangian

$$\mathcal{L}_{\text{int}} = \lambda_{\alpha i} N_\alpha \ell_i H + h_i \bar{e}_i \ell_i H^c + \frac{1}{2} M_\alpha N_\alpha N_\alpha + \text{h.c.} , \quad (1)$$

with summation over repeated indices. The Lagrangian is written in the mass eigenstate basis of RH neutrinos and charged leptons.

For the sake of simplicity, for the rest of the paper we will restrict ourselves to the case in which only the two lightest RH neutrinos are quasi-degenerate, $M_1 \sim M_2 \ll M_3$. We first address the dynamics of the system in the so-called one-flavour approximation, where Boltzmann equations are written for the abundance of the RH neutrinos and for the total lepton asymmetry. This approximation is correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Supposing that

leptogenesis takes place at temperatures $T \sim M_1 \sim M_2$, the one-flavour approximation holds for $M_1 \gtrsim 10^{12}$ GeV. We will include flavour effects later on.

The Boltzmann equations resulting from the CTP formalism, after taking the Markovian limit, are given by [16]

$$\begin{aligned} Y'_N &= -zK \frac{K_1(z)}{K_2(z)} (Y_N - Y_N^{\text{eq}}), \\ Y'_\mathcal{L} &= -2\epsilon(z)Y'_N - \frac{1}{2}Kz^3K_1(z)Y_\mathcal{L}. \end{aligned} \quad (2)$$

Here $Y_N = Y_{N_1} \simeq Y_{N_2}$ denotes the number density of the RH neutrinos per entropy density which we assume to be roughly equal since we also take the decay rates Γ_{N_1} and Γ_{N_2} roughly equal; primes stand for derivatives with respect to the “time” variable $z = M_1/T$ (T being the temperature); the parameter $K \equiv \Gamma_{N_1}/H(M_1) = \Gamma_{N_2}/H(M_1)$, where $H(z)$ is the Hubble rate, controls how much RH neutrinos are out-of-equilibrium; $Y_N^{\text{eq}} = (1/4g_*)z^2K_2(z)$ is the equilibrium number density of RH neutrinos (being g_* the number of relativistic degrees of freedom in the plasma); for consistency, Y_N^{eq} has been computed using the Boltzmann distribution; K_1 and K_2 are the modified Bessel functions of the first and second kind, respectively. In Eq. (2) we have neglected for simplicity the contribution of $\Delta L = 1, 2$ scatterings and thermal effects [2]. Including the contributions of $\Delta L = 1$ scatterings both in the wash-out term and in the CP asymmetry does not change our results significantly. We have summed up the contributions of the two quasi-degenerate RH neutrinos employing the property that $\epsilon_{N_1} = \epsilon_{N_2} \equiv \epsilon$. This follows from having assumed that the decay rates of the two RH neutrinos are nearly equal and therefore both CP asymmetries are resonantly enhanced. If the decay rates are significantly different, then one should pick up only the CP asymmetry contribution which is resonantly enhanced.

Finally, the time-dependent CP asymmetry relevant for resonant leptogenesis is given by [16]

$$\begin{aligned} \epsilon(z) &\simeq \bar{\epsilon} \left[2 \sin^2 \left(\frac{Kz^2}{4} \frac{\Delta M}{\Gamma_{N_2}} \frac{\Gamma_{N_2}}{\Gamma_{N_1}} \right) - \frac{\Gamma_{N_2}}{\Delta M} \sin \left(\frac{Kz^2}{2} \frac{\Delta M}{\Gamma_{N_2}} \frac{\Gamma_{N_2}}{\Gamma_{N_1}} \right) \right], \\ \bar{\epsilon} &= - \frac{\text{Im} \left[(\lambda\lambda^\dagger)_{12}^2 \right]}{(\lambda\lambda^\dagger)_{11} (\lambda\lambda^\dagger)_{22}} \frac{\Delta M/\Gamma_{N_2}}{1 + (\Delta M/\Gamma_{N_2})^2}. \end{aligned} \quad (3)$$

The CP asymmetry therefore consists of two blocks. The first one is the constant piece which is the usually adopted CP asymmetry and is resonantly enhanced for $\Delta M = \Gamma_{N_2}$. It allows efficient generation of the lepton asymmetry even for $M_1 \sim M_2$ as low as the

TeV scale. The other block is made of two oscillating functions. The typical timescale for the variation of the CP asymmetry is

$$t = \frac{1}{2H} = \frac{z^2}{2H(M_1)} = \frac{Kz^2}{2\Gamma_{N_1}} \sim \frac{1}{\Delta M}. \quad (4)$$

The CP asymmetry grows for $t \lesssim 1/\Delta M$ and manifests its oscillation pattern only for $t \gtrsim 1/\Delta M$. The reader familiar with CP violation in neutral meson systems would promptly recognize that the oscillation pattern originates from the CP violating decays of the two mixed states N_1 and N_2 . These states do not propagate freely in the plasma though. If the timescale for the processes relevant for leptogenesis is much larger than $\sim 1/\Delta M$, the CP asymmetry should average to the constant value $\bar{\epsilon}$ quoted in the literature [20, 14, 21]. However, if the timescale of the evolution of the CP asymmetry is larger than or of the order of the timescale of the other processes, the time dependence of the CP asymmetry may not be neglected. This is precisely what happens in the resonant leptogenesis scenario where $\Delta M \sim \Gamma_{N_2} \sim \Gamma_{N_1}$.

Since the strength of the interaction rates is dictated by the parameter K , we expect that in the strong wash-out regime, $K \gg 1$, the effect of the time dependence of the CP asymmetry is negligible. Due to the rapidly oscillating CP asymmetry, the lepton asymmetry should also rapidly oscillate and – at large times – reproduce the value usually quoted in the literature. On the contrary, in the case of weak or mild wash-out, $K \lesssim 1$, the effect of the time dependence of the CP asymmetry should be magnified since the CP asymmetry oscillates with a period comparable to the time scale of the other interactions.

The numerical solutions of the Boltzmann equations support these expectations. Figs. 1 and 2 show the evolution of the lepton asymmetry with and without the time dependence in the CP asymmetry for two representative cases of strong and weak wash-out, respectively. Fig. 3 shows the final baryon asymmetry computed taking into account and neglecting the time dependence in the CP asymmetry as a function of K .

The numerical results can be analytically reproduced. In the strong wash-out regime, $K \gg 1$, $(Y_N - Y_N^{\text{eq}}) \simeq (zK_2(z)/4Kg_*)$ and the lepton asymmetry reads

$$Y_{\mathcal{L}} \simeq \frac{1}{2g_*} \int_0^\infty dz \epsilon(z) z^2 K_1(z) e^{-\frac{K}{2} \int_z^\infty dz' (z')^3 K_1(z')}. \quad (5)$$

Using the steepest descent method, one can easily show that the final lepton asymmetry is equal to the one computed neglecting the time dependence in the CP asymmetry up to

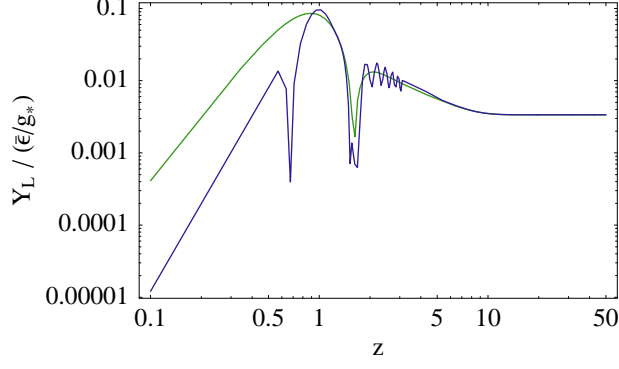


Figure 1. The absolute value of the lepton asymmetry with the time dependence in the CP asymmetry (blue) and without it (green), as a function of z , for $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$ and for $K = 10$.

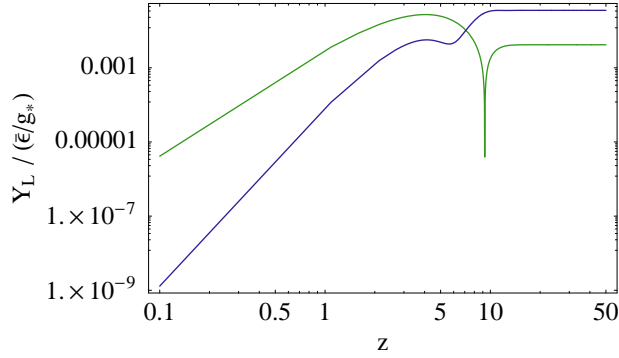


Figure 2. The absolute value of the lepton asymmetry with the time dependence in the CP asymmetry (blue) and without it (green), as a function of z , for $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$ and for $K = 10^{-1}$.

small corrections

$$Y_{\mathcal{L}} \simeq \frac{0.3\bar{\epsilon}}{g_* K^{1.16}} + \mathcal{O}\left(e^{-\frac{3}{2}(\ln K)^2} \frac{1}{K^{1/2}}\right). \quad (6)$$

The weak wash-out regime is much more interesting. Let us first remind the reader what happens in the usual case where the time dependence of the CP asymmetry is neglected. The final lepton asymmetry results from a cancellation between the (anti-) asymmetry generated when RH neutrinos are initially produced and the lepton asymmetry produced when they finally decay. It is useful to define the value $z_{\text{eq}} \gg 1$ as the “time” when the N -abundance reaches the equilibrium abundance: $Y_N(z_{\text{eq}}) = Y_N^{\text{eq}}(z_{\text{eq}})$.

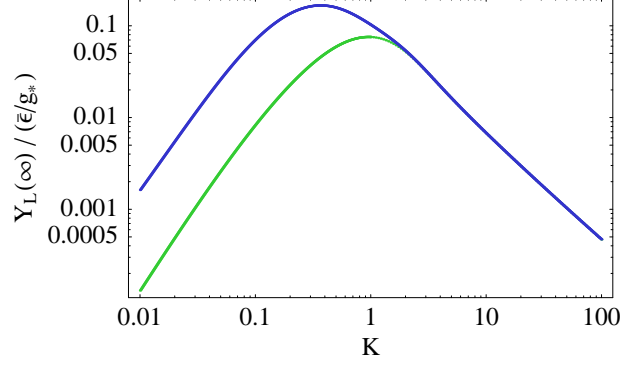


Figure 3. The absolute value of the final lepton asymmetry with the time dependence in the CP asymmetry (blue) and without (green), as a function of K , for $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$.

Since $\int_0^{z_{\text{eq}}} dz' (z')^3 K_1(z') \simeq 3\pi/2$, one finds that z_{eq} is defined implicitly by the relation $z_{\text{eq}}^{3/2} e^{-z_{\text{eq}}} \simeq (3\pi K/2)$. For $z \lesssim z_{\text{eq}}$, inverse decays dominate over decays and $Y_N \ll Y_N^{\text{eq}}$. From Eq. (2) one finds

$$Y_N \simeq K \int_0^z dz' z' \frac{K_1(z')}{K_2(z')} Y_N^{\text{eq}}(z') \simeq \frac{K}{4g_*} \int_0^z dz' (z')^3 K_1(z'). \quad (7)$$

For $z \gtrsim z_{\text{eq}}$, decays dominate over inverse decays, $Y_N \gg Y_N^{\text{eq}}$, and

$$Y_N \simeq Y_N(z_{\text{eq}}) e^{-K \int_{z_{\text{eq}}}^z dz' z' K_1(z')/K_2(z')} \simeq Y_N(z_{\text{eq}}) e^{-K/2(z^2 - z_{\text{eq}}^2)}. \quad (8)$$

The lepton asymmetry is therefore given by

$$\begin{aligned} Y_{\mathcal{L}} &\simeq -\frac{2\bar{\epsilon}K}{4g_*} \int_0^{z_{\text{eq}}} dz' (z')^3 K_1(z') e^{-\frac{K}{2} \int_{z'}^{z_{\text{eq}}} dz'' (z'')^3 K_1(z'')} \\ &\quad - 2\bar{\epsilon} \int_{z_{\text{eq}}}^\infty dz' \frac{d}{dz'} \left(\frac{3\pi K}{8g_*} e^{-K \int_{z_{\text{eq}}}^{z'} dz'' z'' K_1(z'')/K_2(z'')} \right) \\ &\simeq \bar{\epsilon} \left(\frac{1}{g_*} \left(e^{-\frac{3\pi K}{4}} - 1 \right) + \frac{3\pi K}{4g_*} \right) \simeq \frac{\bar{\epsilon}}{g_*} \frac{9\pi^2}{32} K^2. \end{aligned} \quad (9)$$

Let us now come back to the case in which the time dependence of the CP asymmetry is accounted for. This time the near-cancellation between the (anti-) asymmetry generated when RH neutrinos are initially produced and the lepton asymmetry produced when they finally decay is not expected to hold since the asymmetry is modulated by the varying CP asymmetry. Again, we split the final lepton asymmetry as the sum of two contributions

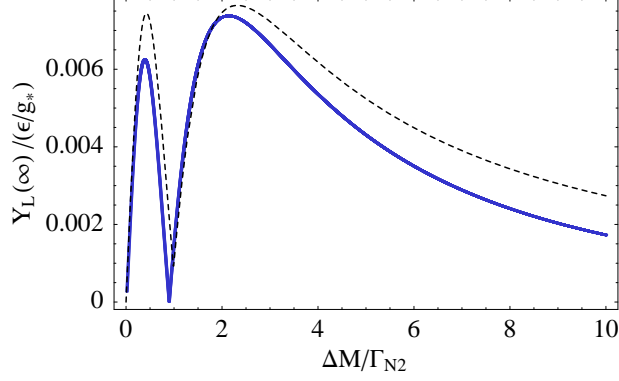


Figure 4. The absolute value of the final lepton asymmetry as a function of $\Delta M/\Gamma_{N_2}$ for $K = 10^{-2}$, $\Gamma_{N_2}/\Gamma_{N_1} = 1$. The blue solid line is obtained by numerically integrating the Boltzmann equations; the black dashed line represents the analytical approximation given in the text. The normalization of the asymmetry by (ϵ/g_*) has been performed only with the part of ϵ which is independent of $\Delta M/\Gamma_{N_2}$, so that the full dependence of $Y_{\mathcal{L}}$ on $\Delta M/\Gamma_{N_2}$ is explicitly shown.

$$\begin{aligned}
Y_{\mathcal{L}} &\simeq \frac{2\bar{\epsilon}K^2}{8g_*} \int_0^{z_{\text{eq}}} dz' (z')^5 K_1(z') + \frac{3\pi K^2}{4g_*} \int_{z_{\text{eq}}}^{\infty} dz' z' \epsilon(z') e^{-K/2(z^2 - z_{\text{eq}}^2)} \\
&\simeq \frac{\bar{\epsilon}}{g_*} \left[\frac{45\pi K^2}{8} + \frac{3\pi K}{4} \frac{1}{1 + (\Delta M/\Gamma_{N_2})^2} \left(1 + \left(\frac{\Delta M}{\Gamma_{N_2}} \right)^2 \right. \right. \\
&\quad \left. \left. - 2 \cos \left[\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}} \right] + \left(\frac{\Delta M}{\Gamma_{N_2}} - \frac{\Gamma_{N_2}}{\Delta M} \right) \sin \left[\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}} \right] \right) \right], \tag{10}
\end{aligned}$$

where in the first contribution we have approximated the CP asymmetry by $\epsilon(z) \simeq -\bar{\epsilon}(Kz^2/2)$. This formula reproduces fairly well the complicated pattern shown in Fig. 4 and shows that the resonance is displaced from the position $\Delta M/\Gamma_{N_2} = 1$ obtained when the time dependence of the CP asymmetry is neglected. Similarly to Eq. (9), the lepton asymmetry scales like K^2 . However, this scaling is not due to cancellations among the asymmetries at different stages, but rather to the fact that for tiny values of K , or equivalently for $t \lesssim 1/\Delta M$, the CP asymmetry is suppressed.

For $\Delta M/\Gamma_{N_2} = 1$, Eq. (10) reduces to

$$Y_{\mathcal{L}} \simeq \frac{\bar{\epsilon}}{g_*} K \left[\frac{45\pi}{8} K + \frac{3\pi}{2} \sin^2 \left(\frac{K z_{\text{eq}}^2}{4} \right) \right]. \tag{11}$$

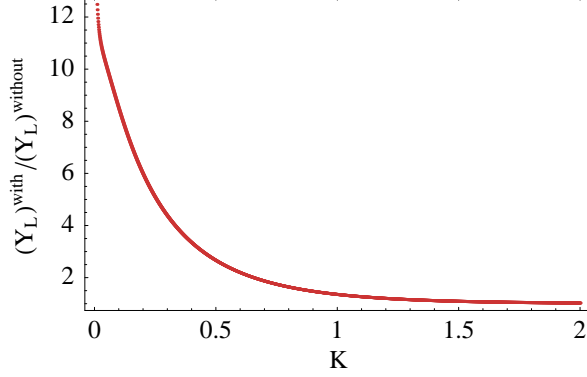


Figure 5. The ratio of the absolute value of the final lepton asymmetry with and without the time dependence in the CP asymmetry as a function of K , for $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$.

For example, for $K = 10^{-2}$, the ratio between the final lepton asymmetries given in Eqs. (11) and (9) is ~ 11 , which is in good agreement with our numerical results, as shown in Fig. 5.

3. Resonant leptogenesis revisited: the flavoured case

Let us now go beyond the one-flavour approximation. As we have already mentioned, it is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Supposing that leptogenesis takes place at temperatures $T \sim M_1 \sim M_2$, the one-flavour approximation only holds for $M_1 \sim M_2 \gtrsim 10^{12}$ GeV. In this range all the interactions mediated by the charged lepton Yukawa couplings are out of equilibrium and there is no notion of flavour. One is allowed to perform a rotation in flavour space to store all the lepton asymmetry in one flavour, the total lepton number. However, at $T \sim 10^{12}$ GeV, the interactions mediated by the charged tau Yukawa come into equilibrium followed by those mediated by the charged muon Yukawa at $T \sim 10^9$ GeV and the notion of flavour becomes physical. Including the issue of flavour can significantly affect the result for the final baryon asymmetry [22, 23, 24, 25, 10, 26, 27]. Thermal leptogenesis is a dynamical process, involving the production and destruction of RH neutrinos and of the lepton asymmetry that is distributed among distinguishable flavours. The processes which wash out lepton number are flavour dependent, *e.g.* the inverse decays from electrons can destroy the lepton asymmetry carried by, and only by,

the electrons. When flavour is accounted for, the final value of the baryon asymmetry is the sum of three contributions. Each term is given by the CP asymmetry in a given flavour properly weighted by a wash-out factor induced by the lepton number violating processes for that flavour.

The Boltzmann equations with flavour taken into account are[‡]

$$\begin{aligned} Y'_N &= -zK \frac{K_1(z)}{K_2(z)} (Y_N - Y_N^{\text{eq}}), \\ Y'_{\ell_i} &= -2\epsilon_i(z)Y'_N - \frac{1}{2}K_i z^3 K_1(z)Y_{\ell_i}, \quad (i = e, \mu, \tau), \end{aligned} \quad (12)$$

where

$$\begin{aligned} \epsilon_i(z) &\simeq \bar{\epsilon}_i \left[2 \sin^2 \left(\frac{Kz^2}{4} \frac{\Delta M}{\Gamma_{N_2}} \frac{\Gamma_{N_2}}{\Gamma_{N_1}} \right) - \frac{\Gamma_{N_2}}{\Delta M} \sin \left(\frac{Kz^2}{2} \frac{\Delta M}{\Gamma_{N_2}} \frac{\Gamma_{N_2}}{\Gamma_{N_1}} \right) \right], \\ \bar{\epsilon}_i &= - \frac{\sum_{j=e,\mu,\tau} \text{Im} \left(\lambda_{1i} \lambda_{1j} \lambda_{j2}^\dagger \lambda_{i2}^\dagger \right)}{(\lambda \lambda^\dagger)_{11} (\lambda \lambda^\dagger)_{22}} \frac{\Delta M / \Gamma_{N_2}}{1 + (\Delta M / \Gamma_{N_2})^2}, \\ K_i &\equiv \frac{\Gamma(N_1 \rightarrow \ell_i H)}{H(M_1)} \simeq \frac{\Gamma(N_2 \rightarrow \ell_i H)}{H(M_1)}, \quad K = \sum_{j=e,\mu,\tau} K_j. \end{aligned} \quad (13)$$

Notice that, for simplicity, we have again assumed that the partial rates for the decay of the two quasi-degenerate RH neutrinos are nearly equal. Extending our study to the case where the rates are different is straightforward.

What we have learned so far is that significant differences with respect to the case in which the CP asymmetries are constant in time are expected in the weak wash-out regime. Let us then suppose that all flavours are in the weak wash-out regime and also that $K \lesssim 1$. If the time dependence of the CP asymmetry is neglected each individual flavour asymmetry suffers the usual cancellation and is given by $Y_{\ell_i} \simeq 2.8(\bar{\epsilon}_i/g_*)KK_i$ [26]. Solving the Boltzmann equations along the same lines leading to the results (10) and (11) shows that flavour asymmetries, when the time dependence in ϵ_i is taken into account, are given by (for $\Delta M/\Gamma_{N_2} = 1$)

$$Y_{\ell_i} \simeq \frac{\bar{\epsilon}_i}{g_*} K \left[\frac{45\pi}{8} K + \frac{3\pi}{2} \sin^2 \left(\frac{Kz_{\text{eq}}^2}{4} \right) \right]. \quad (14)$$

Therefore, the flavour asymmetries are increased by a factor $\sim 10(K/K_i)$ compared to

[‡] We neglect both quantum flavour correlations [10] and the corrections arising from connecting the asymmetries in the lepton doublets to the asymmetries in the charges $\Delta_i = B/3 - L_i$ conserved by weak sphalerons [22, 10, 25, 26]. They introduce small corrections to our results.

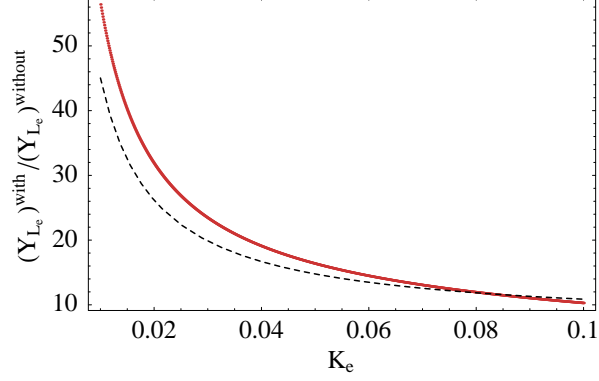


Figure 6. The ratio of the absolute value of the final lepton asymmetry in the e flavour, with and without the time dependence in the CP asymmetry, as a function of K_e , for $K_\mu = 0.05$, $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$. The red solid line is obtained by numerically integrating the Boltzmann equations; the black dashed line represents the analytical approximation given in the text.

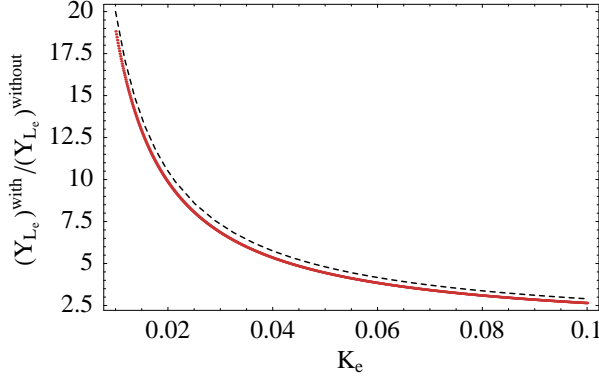


Figure 7. The ratio of the absolute value of the final lepton asymmetry in the e flavour, with and without the time dependence in the CP asymmetry, as a function of K_e , for $K_\mu = 10$, $\Delta M/\Gamma_{N_2} = \Gamma_{N_2}/\Gamma_{N_1} = 1$. The red solid line is obtained by numerically integrating the Boltzmann equations; the black dashed line represents the analytical approximation given in the text.

what is obtained when the CP asymmetry is assumed to be a constant. This result is confirmed by the numerical study shown in Fig. 6.

Let us now consider the case in which some flavour is in the strong wash-out regime, so that $K \gg 1$, but some flavour i is weakly coupled, $K_i \lesssim 1$. Notice that this case is peculiar to the scenario where flavour is taken into account; it has no analogue in the one-flavour regime. Now the lepton asymmetry stored in the flavour i without accounting

for the time-dependence of the CP asymmetry reads [26]

$$\begin{aligned} Y_{\ell_i} &\simeq -2\bar{\epsilon}_i \int_0^\infty dz Y'_N e^{-\frac{K_i}{2} \int_z^\infty dz' (z')^3 K_1(z')} \\ &= \bar{\epsilon}_i K_i \int_0^\infty dz Y_N^{\text{eq}} z^3 K_1(z) \simeq 0.8 \frac{\bar{\epsilon}_i K_i}{g_*} \end{aligned} \quad (15)$$

where we have made use of the fact that, for $K \gg 1$, $Y_N \simeq Y_N^{\text{eq}}$. The result is proportional to the weak wash-out parameter K_i because, in the limit of vanishing K_i , the asymmetry produced by inverse decays is cancelled by the one generated by the decays of the RH neutrinos. If the time dependence of the CP asymmetry is accounted for, the lepton asymmetry in the flavour i is given by

$$\begin{aligned} Y_{\ell_i} &\simeq -2 \int_0^\infty dz \epsilon_i(z) Y'_N e^{-\frac{K_i}{2} \int_z^\infty dz' (z')^3 K_1(z')} \\ &\simeq 2 \int_0^\infty dz \epsilon'_i(z) Y_N + K_i \int_0^\infty dz \epsilon_i(z) Y_N z^3 K_1(z). \end{aligned} \quad (16)$$

where in the second line we have neglected the damping factor. The piece proportional to the derivative of the CP asymmetry may be evaluated as follows. Since $K \gg 1$, the RH abundance can be approximated as

$$Y_N(z) \simeq \begin{cases} \frac{1}{2g_*} \left(1 - e^{-\frac{K}{6} z^3}\right) & (z < z_{\text{eq}}), \\ \frac{1}{4g_*} z^2 K_2(z) & (z > z_{\text{eq}}), \end{cases} \quad (17)$$

where $z_{\text{eq}} \simeq (6/K)^{1/3}$. The integral can now be evaluated as the sum of three different pieces, for $0 < z < z_{\text{eq}}$, $z_{\text{eq}} < z < 1$ and for $z > 1$. The latter is negligible since for large K the oscillating functions in the derivative of the CP asymmetry average to zero. The other two integrals can be easily evaluated using the expression (17) and give

$$2 \int_0^\infty dz \epsilon'_i(z) Y_N \simeq -\frac{\bar{\epsilon}_i}{2g_*} z_{\text{eq}} \frac{\Gamma_{N_2}}{\Delta M} \left[\frac{\Gamma_{N_2}}{\Delta M} \cos\left(\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}}\right) - \sin\left(\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}}\right) \right]. \quad (18)$$

The final lepton asymmetry in the flavour i is therefore

$$Y_{\ell_i} \simeq 0.8 \frac{\bar{\epsilon}_i}{g_*} K_i - \frac{\bar{\epsilon}_i}{2g_*} z_{\text{eq}} \frac{\Gamma_{N_2}}{\Delta M} \left[\frac{\Gamma_{N_2}}{\Delta M} \cos\left(\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}}\right) - \sin\left(\frac{K z_{\text{eq}}^2 \Delta M}{2 \Gamma_{N_2}}\right) \right]. \quad (19)$$

For instance, for $K = 10$ and $\Delta M \sim \Gamma_{N_2}$, one obtains $z_{\text{eq}} \simeq 0.85$ and the ratio between the asymmetries (19) and (15) goes like $1 + (0.19/K_i)$, which agrees with the numerical results, as shown in Fig. 7.

4. Conclusions

Resonant leptogenesis has received much attention since it allows an efficient generation of the baryon asymmetry for RH neutrinos as light as the TeV scale. Through a non-equilibrium quantum field theory approach to leptogenesis, we have recently shown that the CP asymmetry parameter is not constant in time, but it varies with a typical timescale equal to the mass difference of the RH neutrinos [16]. In resonant leptogenesis, the two RH neutrinos N_1 and N_2 are almost degenerate in mass and the CP asymmetry from the decay of the first RH neutrino N_1 is resonantly enhanced if the mass difference $\Delta M = (M_2 - M_1)$ is of the order of the decay rate of the RH neutrinos. Therefore, the typical timescale of variation of the CP asymmetry can be larger than the timescale for the change of the abundances of the RH neutrinos.

We have studied what differences arise with respect to the case in which the CP asymmetry is assumed to be a constant. Let us summarize our results:

1) One-flavour case, valid for $M_1 \sim M_2 \gtrsim 10^{12}$ GeV: the expression for the final baryon asymmetry differs from the results appeared so far in the literature in the weak and mild wash-out regime, $K \lesssim 1$. The baryon asymmetry scales like K^2 , see Eqs. (10) and (11), and is a factor $\mathcal{O}(10)$ larger than the baryon asymmetry computed with constant CP asymmetry.

2) Flavoured case, to be applied when $M_1 \sim M_2 \lesssim 10^{12}$ GeV: the expression for the final baryon asymmetry differs from the results appeared so far in the literature for those flavours i which are in the weak and mild wash-out regime. If $K \lesssim 1$ and $K_i \lesssim 1$, the asymmetry in the flavour i scales like K^2 , see Eq. (14), and is enhanced by a factor $\sim 10(K/K_i)$ with respect to the case in which the CP asymmetry is constant. If $K \gtrsim 1$ and $K_i \lesssim 1$, the asymmetry in the flavour i is given by Eq. (19) and can be enhanced by a factor proportional to $1/K_i$ with respect to the case in which the CP asymmetry is constant.

We conclude that the memory effects encoded in the time-dependent CP asymmetry may play an important role in resonant leptogenesis.

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